Your Signature ________________________

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Please write your name on every page of this booklet and every additional sheet taken.

3. If you are using a Theorem/Result from class please state the result clearly and verify the hypotheses of the same.

4. **Maximum time is 1.5 hours and Maximum Possible Score is 50.**

Score

Q.No.	**Alloted Score**	**Score**
1.	10	
2.	15	
3.	15	
4.	15	
Total	55	

Number of Extra sheets attached to the answer script: ________________________

1. Suppose $X \sim$ Uniform $(0, 2\pi)$ random variable on a probability space $(\Omega, \mathcal{B}, \mathbb{P})$. Let $Y = \tan(X)$. Find the distribution function and then the probability density function of Y.

2. Let X_n be a sequence of independent random variables on $(\Omega, \mathcal{B}, \mathbb{P})$. Consider for $t \in \mathbb{R}$, the random power series

$$\sum_{n=1}^{\infty} X_n t^n.$$

Show that there is a $t_0 > 0$ such that for $\mid t \mid < t_0$ the random power series converges with probability 1.

3. Let X_n be a sequence of random variables on a probability space $(\Omega, \mathcal{B}, \mathbb{P})$, such that

$$\mathbb{P}(X_n = x) = \begin{cases} \frac{1}{2^n} & \text{if } x = 1 \\ \frac{1}{4^n} & \text{if } x = 2 \\ 1 - \frac{1}{4^n} - \frac{1}{2^n} & \text{if } x = 3 \end{cases}$$

Let $A_n = \{X_n = 3\}$. Find $\mathbb{P}(\limsup_{n\to\infty} A_n)$.

4. Consider $([0,1],\mathcal{B},\mathbb{P})$ with $\mathbb{P}$ being the uniform measure on $[0,1]$. Give examples of a random variable X defined on this probability space such that

(a) $\mathbb{E}[X^+] = \infty$ and $0 < \mathbb{E}[X^-] < \infty$.

(b) $\mathbb{E}[X^+] = \mathbb{E}[X^-] = \infty$.

(c) $\mathbb{E}[X] < \infty$ and $\mathbb{E}[X^2] = \infty$.